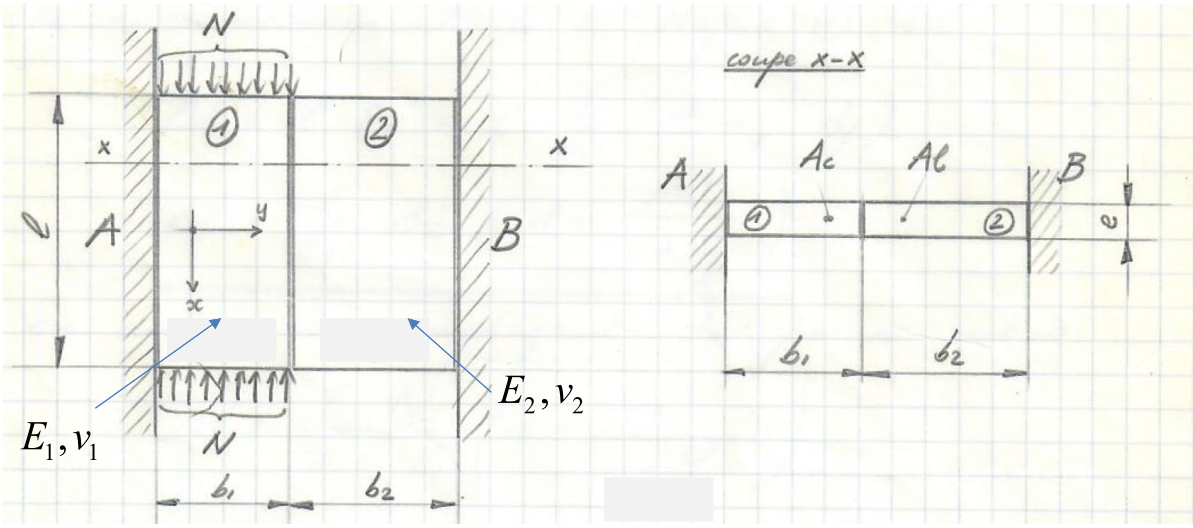
Exercise 1: A plate of steel and a plate of aluminum of the same length l are placed between two vertical rigid supports A and B. Assuming that the friction between them is negligible, calculate the decrease in length Δl of the steel plate,



Solution:

Stresses on the plates. The force N produces

1. a vertical $\sigma_x = \dfrac{N}{b_1 e}$ and a horizontal stress σ_y on the steel plate along the contact zone
2. a horizontal stress σ_y along the contact zone of the two plates.

Deformation.

Since the supports are rigid, we have the following condition,

$$\Delta b_1 + \Delta b_2 = 0 . \qquad \text{(a)}$$

Due to geometry and loading, each plate is in plane stress:

For the steel plate

$$\varepsilon_x = \frac{1}{E_1}\left(\sigma_x - \nu_1 \sigma_y\right), \quad \varepsilon_y = \frac{1}{E_1}\left(\sigma_y - \nu_1 \sigma_x\right). \qquad \text{(b)}$$

$$\Delta b_1 = b_1 \varepsilon_y = \frac{b_1}{E_1}\left(\sigma_y - \nu_1 \sigma_x\right)$$

For the aluminum plate,

$$\varepsilon_x = \frac{1}{E_2}\left(-\nu_2 \sigma_y\right), \quad \varepsilon_y = \frac{1}{E_2}\left(\sigma_y\right). \qquad \text{(c)}$$

$$\Delta b_2 = b_2 \varepsilon_y = \frac{b_2}{E_2}\left(\sigma_y\right).$$

From (a) we have $\frac{b_1}{E_1}\left(\sigma_y - v_1 \sigma_x\right) + \frac{b_2}{E_2}\left(\sigma_y\right) = 0 \quad \Rightarrow \quad \sigma_y - v_1 \sigma_x + \frac{b_2}{b_1}\frac{E_1}{E_2}\sigma_y = 0$

Define $\varphi = \frac{b_2}{b_1}; \quad \lambda = \frac{E_2}{E_1} \quad \Rightarrow \sigma_y = \sigma_x \frac{v_1}{1+\varphi/\lambda}$ (d)

For the steel plate,

$$\Delta \ell = \ell \varepsilon_x = \ell \frac{1}{E_1}\left(\sigma_x - v_1 \sigma_y\right) \tag{e}$$

Inserting (c) in (d) we obtain, $\Delta \ell = \frac{\ell}{E_1}\frac{N}{b_1 e}\left(1 - \frac{v_1^2}{1+\varphi/\lambda}\right)$

Exercise 2: A steel tank with an internal diameter radius of $2r_i =$ 1.4 m is subjected to an internal pressure $p_i = 8$MPa. The tensile and compressive elastic limits are σ_{yp} = 240 MPa. What is the wall thickness with a safety factor of 2?

Solution

From the stress analysis, the maximum stress is at the inter surface and given by,

$$\sigma_{\theta\theta,\max} = \sigma_{\theta\theta}\Big|_{r=r_i} = \frac{r_i^2 P_i}{r_e^2 - r_i^2}\left[1 + \frac{r_e^2}{r_i^2}\right] = P_i \frac{r_i^2 + r_e^2}{r_e^2 - r_i^2}$$

From this equation we obtain,

$$r_e = \left(\frac{r_i^2 (p_i + \sigma_{\theta\theta,\max})}{\sigma_{\theta\theta,\max} - p_i}\right)^{1/2}$$

Substituting the numerical values we obtain, $r_e = 0.748$m. The wall thickness is 0.748-0.700 = 0.048m.

Exercise 3: Demonstrate that for an annular rotating disk the ratio of the maximum tangential stress to the maximum radial stress is (v is the Poisson ratio),

$$\frac{\sigma_{\theta\theta,\max}}{\sigma_{rr,\max}} = \frac{2}{\left(r_e - r_i\right)^2}\left(r_e^2 + \frac{1-v}{3+v} r_i^2\right)$$

Solution

Stress analysis gave the following results for the two stress components:

$$\sigma_{rr} = \frac{(3+v)}{8}\left(r_i^2 + r_e^2 - r^2 - \frac{r_i^2 r_e^2}{r^2}\right)\rho\omega^2$$

$$\sigma_{\theta\theta} = \frac{(3+v)}{8}\left(r_i^2 + r_e^2 - \frac{1+3v}{3+v} r^2 + \frac{r_i^2 r_e^2}{r^2}\right)\rho\omega^2$$

The maximum values of these two stresses are at $r=(r_i r_e)^{1/2}$ and $r = r_i$, respectively. Inserting these values in the preceding equations it is easy to show that,

$$\sigma_{rr}\Big|_{r=\sqrt{r_i r_e}} = \frac{(3+v)}{8}\left(r_i^2 + r_e^2 - r_i r - \frac{r_i^2 r_e^2}{r_i r_e}\right)\rho\omega^2 = \frac{(3+v)}{8}\left(r_i - r_e\right)^2 \rho\omega^2$$

$$\sigma_{\theta\theta}\Big|_{r=r_i} = \frac{(3+v)}{8}\left(r_i^2 + r_e^2 - \frac{1+3v}{3+v} r_i^2 + \frac{r_i^2 r_e^2}{r_i^2}\right)\rho\omega^2 = \frac{2(3+v)}{8}\left(r_e^2 + \frac{1-v}{3+v} r_i^2\right)\rho\omega^2$$

Thus, their ratio gives the result.

Exercise 4: Calculate the allowable angular rotation in rpm of a flat solid disk with radius r_e = 125 mm. The disk is made of an aluminum alloy with σ_{yp} = 280MPa, Poisson ratio 1/3 and density ρ=2.7kNs^2/m^4. Use the maximum distortion energy criterion.

Solution

In a solid disk the stresses are maxima at the origin $r = 0$.:

$$\sigma_{rr}\big|_{r=0} = \frac{(3+v)}{8}\left(r_e^2 - r^2\right)\rho\omega^2 = \frac{(3+v)r_e^2}{8}\rho\omega^2$$

$$\sigma_{\theta\theta}\big|_{r=0} = \frac{(3+v)}{8}\left(r_e^2 - \frac{(1+3v)}{3+v}r^2\right)\rho\omega^2 = \frac{(3+v)r_e^2}{8}\rho\omega^2$$

$$\Rightarrow \sigma_{rr} = \sigma_{\theta\theta}$$

The distortion energy criterion is

$$\sigma_{yp} = \frac{1}{\sqrt{2}}\left[\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2\right]$$
$$= \frac{1}{\sqrt{2}}\left[\left(\sigma_{rr} - \sigma_{\theta\theta}\right)^2 + \left(\sigma_{\theta\theta} - 0\right)^2 + \left(0 - \sigma_{rr}\right)^2\right] = \sigma_{rr} = \sigma_{\theta\theta}$$

$$\sigma_{yp} = \frac{(3+v)r_e^2}{8}\omega^2 \quad \Rightarrow \quad \omega = \frac{1}{r_e}\sqrt{\frac{8\sigma_{yp}}{(3+v)\rho}}$$

We insert the numerical values to obtain $\omega = \frac{1}{0.125}\sqrt{\frac{8\cdot 280\cdot 10^6}{(3+1/3)2.7\cdot 10^3}} = 3991.1$ rad/sec.

Thus, $\omega = (3991.1)60/2\pi = 38{,}131.5$ rpm.